Intrinsic functionality of mathematics, metafunctions in Systemic Functional Semiotics

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1 Mathematics and language

'Mathematics is a language'. Such a statement commonly arises in common sense conversations about the nature of mathematics in relation to language and regularly sparks lively conversation in academic circles. Although at first glance there are significant differences between the two, as Danesi (2016: xi) suggests, such a characterisation may not be just descriptive or figurative; mathematics and language share many structural properties. Like every language, mathematics maintains its own coherent grammatical organisation and presents its own set of symbols. Indeed this similarity has led in part to language being modeled according to mathematical principles in many branches of linguistics. In this sense, there is much that suggests we can group them as broadly the same. However despite this, it is obvious that mathematics and language are used for different purposes and occur in different social situations. Mathematics is primarily written, its vast symbolism tends to occur only in intellectualised contexts, it doesn't allow for dialogue or negotiation and it regularly occurs alongside 'other' languages, suggesting it performs a different function. In this way, suggesting that saying mathematics is a language seems as though it may just be, in Danesi's words, figurative.

Rather than a simple designation of mathematics as a language or not, a more fruitful approach is likely to arise from a principled comparison of various senses in which the two are similar or different. In this way, we can understand why each is used for different purposes and develop a more coherent picture of each system's functionality. To explore this issue, this paper will utilise broad theoretical tools associated with the multimodal semiotic approaches of Systemic Functional Semiotics and Social Semiotics (Hodge and Kress 1988; O'Toole 1990; van Leeuwen 2005; Martin 2011). In doing so, it will raise issues for how these approaches model the intrinsic functionality of various semiotic systems and from this, how semiotic descriptions are developed.

One of the guiding principles underpinning much of the field of multimodal semiotics is that different semiotic resources have different functionalities for different situations. This principle has driven much research to focus on how different semiotic resources work, how these resources are organised and from this, why they are used. The Social Semiotic and Systemic Functional tradition investigating these questions has tended to take as its point of departure the twin models of images by Kress and van Leeuwen (1990, 2006) and O'Toole (1990, 1994). These descriptions laid out a number of the premises that have characterised the field since and have underpinned the manner through which many semiotic systems have been investigated. In particular, both descriptions took inspiration from Halliday's (1978) conception of language as social semiotic. This involves interpreting language and broader semiosis as a resource for meaning that is intricately connected to the social context in which it is used. A major part of Halliday's (1978) conceptualisation of language as social semiotic is that language has an intrinsic functionality that organises its overall architecture. Under this model, this functionality distributes features of language into a series of functional components known as metafunctions that are relatively independent of each other and are realised through distinct structures. These metafunctions have become the means through which Halliday and the resulting Systemic Functional and Social Semiotic traditions have conceptualised the interplay between the social functions of language and its internal organisation.

In language, three main metafunctions are distinguished. The ideational metafunction is roughly glossed as construing 'reality' and involves systems such as the TRANSITIVITY of the English clause (distinguishing between, for example, relational clauses often shown through the verb 'to be' as in, *I am human*, and mental clauses often shown through verbs of perception, thought, desideration etc. such as in, *I need to be loved*). The interpersonal metafunction enacts sociality through resources for negotiation in dialogue and evaluative stance through systems such as MOOD and MODALITY (distinguishing between, for example, the declarative *the leather runs smooth on the passenger seat*, and the interrogative *why pamper life's complexity?*). The textual metafunction manifests text, texture and information flow through systems such as THEME (distinguishing, for example, the marked theme of *in my heart it was so real* from the unmarked theme in *it was so real in my heart*) (Halliday and Matthiessen 2014). These three metafunctions are generally understood to be broadly independent of each other, with relatively free choice of combination between the systems (Halliday 1978; Martin 1991). In addition to this three-way division, within the ideational metafunction, a common distinction is made between the logical and experiential components that separate the recursive ideational systems (logical) from those that are not (experiential). This three-way distinction of metafunction (or four-way if distinguishing the logical and experiential) has underpinned Systemic Functional conceptions of language since its introduction in the late 1960s (Halliday 1969).

At the advent of the Social Semiotic approach to multimodality, Kress and van Leeuwen (1990, 2006: 42) argued that these metafunctions "apply to all semiotic modes, and are not specific to speech and writing". In this they were accompanied by O'Toole's (1990, 1994) model of visual art that described metafunctions as "three universal functions" (though renamed as representational, modal and compositional functions, 1990: 187). Accordingly, one of the main starting points for both Kress and van Leeuwen's and O'Toole's grammars of images became the ideational, interpersonal and textual metafunctions themselves.

Following these accounts, the Social Semiotic and Systemic Functional traditions have tended to follow this generalisation and treat descriptions of semiosis as being organised metafunctionally. Examples of this include models of mathematical symbolism (O'Halloran 1999, 2005), bodily action (Martinec 1998, 2000, 2001), children's picture books (Painter et al. 2013), three-dimensional space (Ravelli and McMurtrie 2016) and numerous others. Although there have been some notable exceptions to this assumption such as van Leeuwen's model of sound (1999) and Halliday's model of child proto-language (1975), a three or four-way metafunctional division has tended to permeate the field's understanding of semiosis.

Beginning with metafunctions has allowed semiotic work to see the meanings of simultaneous variables, without some being backgrounded for others. For example, Kress and van Leeuwen's work on images allowed an understanding of the conventional means through which activities and taxomonies are organised in images, while also showing the variations in interpersonal modality and power-relations, and their organisation for informational prominence all at the same time. In addition, models based on metafunctionality have offered an avenue through which the systems of each resource can be interpreted as meaning rather than simply form. However more practically, beginning with metafunctions opened an avenue through which a large community associated with Systemic Functional Linguistics could get a handle on resources outside of language and compare and contrast them on comparable terms. In this way, assumptions of metafunctions have proven remarkably productive and beneficial for expanding the frontier of Social Semiotic and Systemic Functional theory.

However the assumption of metafunctions across semiosis raises a number of issues. These largely stem from the fact that metafunctions were initially derived from descriptions of a small handful of languages (primarily English, Halliday 1969) and have tended to be carried over to other semiotic resources unquestioningly. This is problematic if we wish to build descriptions that bring out the specific functionality of each semiotic resource. If we take metafunctionality to be the one of the broadest means by which these traditions conceptualise the intrinsic functionality of semiotic resources, by simply assuming metafunctions across semiosis, we run the risk of homogenising descriptions and making everything look like the first resource to be comprehensively described (i.e. English). That is, we risk watering down the specific functionality of each resource.

For the purpose of understanding mathematics in relation to language, this means that an assumption of the same metafunctional configuration for both resources will inevitably lead to them both looking relatively similar, diluting any attempt to investigate the broad distinctions in their functionality. Thus with this as its background, this paper argues that Systemic Functional and Social Semiotic studies cannot simply assume metafunctions will occur across every resource. These categories were originally developed for language and as yet, there has not been a detailed justification for their use in other resources. This is not to say they won't occur, but that for such categories to be used in the description of a semiotic resource, they need to be justified internally to the system being studied. For mathematical symbolism, this means that for a model of metafunctions to be productive, it needs to be derived from its internal grammatical organisation. Once this is done, we are able to compare mathematics and language in a more principled nature and see that although they share a number of broad characteristics, there are also large functional dimensions that significantly diverge.

2 The descriptive basis for metafunction

Such an interpretation of metafunction raises an important question. If we cannot assume metafunctions in the Social Semiotic/Systemic Functional approach, what can we assume?

Put more broadly, what is the basis for constructing descriptions? Martin (2013) argues that metafunctions (and indeed all systemic functional macrotheoretical categories such as rank and strata) can be derived from the more fundamental theoretical primitive of axis - the interaction of the paradigmatic and syntagmatic axes (also less formally known as system and structure, and formalised in system networks). These axes are the foundational principles of Systemic Functional theory that evolved from Saussure (1916), Hjelmslev (1943) and Firth (1957, 1968). The ability of metafunction and other macrotheoretical categories to be derived from axis opens a powerful method through which the architecture of various semiotic resources can be developed and justified. Before moving on to mathematical symbolism, we will illustrate how metafunctions are derived from axis in language. This discussion primarily draws upon that of Martin (1983 and 2013).

The justification for metafunctions is based on two types of evidence: first, the relative paradigmatic in(ter)dependence of systems (originally developed through bundling of clausal systems in English, Halliday 1967a, 1967b, 1968, 1969, 1970), and second, the types of syntagmatic structure (Halliday 1979; Martin 1983). In terms of paradigmatic systems, the basis for suggesting distinct metafunctional components hinges on systems being more or less independent of each other. If choices in a bundle of systems are shown to be heavily dependent on other choices in the bundle then it suggests that these systems are all part of the same metafunction. However, if this bundle of systems is relatively independent of another bundle of systems, it suggests that they form different metafunctional components. In relation to the English clause, an example of this is shown through the distinction between TRANSITIVITY and MOOD. TRANSITIVITY and MOOD are relatively independent of each other, which means that any choice in TRANSITIVITY has relatively free choice of MOOD, with only a few constraints. This is shown in Table 1.[1]

[1] As we are developing a systemic argument, Matthiessen's (1995) description of nuclear transitivity is being followed here – it being the most fully developed paradigmatic account. In this description, the four least delicate process types are material, mental, verbal and relational. Existential and behaviourals, considered to be at primary delicacy in Halliday and Matthiessen (2014), are taken in Matthiessen's model as more delicate subtypes of relational and material, respectively.

Table 1 MOOD vs TRANSITIVITY in English

MOOD / TRANSITIVITY	**indicative**	**imperative**
material	*Cronulla won the premiership.*	*Win the premiership.*
mental	*It pleased the fans.*	*Please the fans.*
relational	*She is a good captain.*	*Be a good captain.*
verbal	*She praised the fans.*	*Praise the fans.*

The relative independence of MOOD choices with TRANSITIVITY indicates their potential to be part of distinct functional components that form the basis of metafunctions. In contrast, comparing the relation between MOOD and MODALITY shows that the MODALITY system is entirely dependent on choices within the MOOD system (Martin 2013: 52). In particular, choices in MODALITY can only be made if indicative and not imperative is chosen in MOOD (the asterisk * indicates that an example is not possible).

(1) *They may cook me a tart* indicative + modality

(2) **may cook me a tart* imperative + modality

In contrast, like MOOD, modality can occur for all TRANSITIVITY types:

(3) *They may cook me a tart* material + modality

(4) *It may please my stomach* mental + modality

(5) *She may be a good cook* relational + modality

(6) *I may praise her* verbal + modality

Paradigmatically, therefore, MOOD and MODALITY are interdependent. This suggests that they are organised within the same functional component. In addition, MOOD and MODALITY are both independent of TRANSITIVITY, suggesting that they form a distinct functional component from it.

The second branch of axial evidence for metafunction involves the type of syntagmatic structure used to realise systemic choices. This draws on Halliday's (1979) suggestion that distinct functional components in language tend to be realised by different modes of meaning (influenced by Pike's 1959 suggestion that language can be looked at as particles, waves or fields). Halliday argues that ideational meanings are realised through particulate structures where each element is sharply bounded from each other. Within the ideational metafunction, the experiential component involves *multivariate* particulate structures, where each element performs a distinct function that generally only occurs once. On the other hand, the logical component involves *univariate* particulate structures where a single function is repeated indefinitely. An example of an experiential multivariate structure is the TRANSITIVITY functions of the English clause: *We* (Carrier) *are* (Process) *all good* (Attribute). Each of the elements perform distinct functions (in brackets) and typically occur only once. An example of a logical univariate structure is the complexing of nominal groups that can function as a single participant in *Me* (1) *and Frank* (2) *and my brother* (3). In this example, each element performs the same function, related through coordination, with any number of elements allowed to be coordinated with them.

In contrast to the particulate structures of the ideational component, the textual metafunction is organised through a periodic (or wave-like) function. In the English clause, for example, there are two positions of informational prominence that typically occur at the beginning (the Theme) and the end (New) of the clause, such as in *Tomorrow I* (Theme)*'ll meet you at the cemetery gates* (New). These two elements indicate relative salience in the information flow in relation to elements that have been backgrounded (known as Rheme and Given). With each clause in a text showing a similar variation in salience, the text presents a periodic organisation of informational prominence that links up with higher levels of prominence in discourse (known as hyper/macroThemes and News, Martin 1992) and lower level phonological/graphological levels of prominence in intonation, feet and syllable structure (Halliday 1992a; Cléirigh 1998). Importantly, the possibility for informational prominence at various levels builds a hierarchy of prominence that can extend across an indefinite length of text.

Finally, interpersonal systems are realised through prosodic structures that cut across constituent elements. An example of this is the choice of negative polarity in English, which affects indefinite deixis throughout the clause. This is most easily seen in a non-standard English dialect that gives *I* <u>*ain't*</u> *got* <u>*no*</u> *money* <u>*no*</u>*where* (though it is just as regular in the standard English version of *I don't have any money anywhere*; contrast with the positive, *I do have some money somewhere*). These metafunctional components and their types of structure are shown in Table 2.

Table 2 Metafunctions and structure in language

Metafunction	Structure	Example
ideational - experiential	particulate: - multivariate	*We* / *are* / *in our bedroom* Carrier / Process / Location
- logical	- univariate	*Me* / *and Frank* / *and my brother* 1 / 2 / 3
textual	periodic	*Tomorrow* / *'ll meet* / *at the cemetery* *I* / *you* / *gates* **Theme** ⟶ Rheme Given ⟵ **New**
interpersonal	prosodic	*I ain't got no money nowhere* neg ⟶

As MOOD and MODALITY are considered interpersonal systems, this line of argumentation concerns their similarity of prosodic structure. Beginning with MOOD, distinctions between relatively indelicate types of MOOD (imperative, declarative, interrogative etc.) in English revolve around the presence or absence of the functions Subject and Finite. Indicative clauses have both a Subject and a Finite, where imperative clauses typically have neither. The subtypes of indicative, declarative and interrogative differ in terms of their ordering of the two functions: Subject before Finite for declarative, and Finite before Subject for interrogative, as in (with Subject and Finite in bold):

(7) *Take them home* — imperative (no Subject or Finite)

(8) ***Will you** take them home?* — interrogative (Finite before Subject)

(9) ***You will** take them home.* — declarative (Subject before Finite)

The Subject and Finite are the crucial realisation functions for MOOD types in English. Looking syntagmatically, these functions also present a prosody of PERSON, NUMBER, GENDER and TENSE or MODALITY through the rest of the clause (more commonly known as 'agreement' or 'concord'). For example, the Subject and Finite must agree in terms of NUMBER and PERSON:

(10)	***Am I*** *taking them home?*	first person, singular
(11)	***Are you*** *taking them home?*	second person, singular
(12)	***Is he/she*** *taking them home?*	third person, singular
(13)	***Are we/you/they*** *taking them home?*	first/second/third person, plural

In addition, the Subject and Finite must agree with their counterparts in the Moodtag in terms of PERSON, NUMBER, GENDER and either TENSE or MODALITY.

(14) ***I am*** *taking them home,* ***aren't I****?*	first person, singular, present
(15) ***You are*** *taking them home,* ***aren't you****?*	second person, singular, present
(16) ***He is*** *taking them home,* ***isn't he****?*	third person, singular, masculine, present
(17) ***She is*** *taking them home,* ***isn't she****?*	third person, singular, feminine, present
(18) ***She will*** *take them home,* ***won't she****?*	third person, singular, feminine, future
(19) ***She was*** *taking them home,* ***wasn't she?***	third person, singular, feminine, past

(20) ***She must*** *take them home,* ***mustn't she?***	third person, singular, feminine, modal: high
(21) ***She can*** *take them home,* ***can't she?***	third person, singular, feminine, modal: low

This agreement between the central interpersonal functions of Subject and Finite, and the Moodtag (also considered an interpersonal system, Martin 2013: 59) can be interpreted as a structural prosody encompassing the breadth of the clause. The choice of PERSON, NUMBER etc. affects multiple elements across the entire clause.

Similarly, MODALITY also displays a prosodic structure. This can be most clearly seen when discussing metaphors of modality, such as the example given by Halliday (1979: 66):

(22) *I wonder if perhaps it might be measles, might it d'you think?*

In this example, the same modality choice is realised a number times through different expressions: *I wonder*, *perhaps*, *might*, *might* and *d'you think*. As Halliday suggests, each could work effectively on its own, but working together the effect is cumulative, reasserting the speaker's angle on the statement and giving a prosody that colours the entire clause.

Following Martin's (1983, 2013) argumentation, as both MOOD and MODALITY have a large degree of paradigmatic interdependence and are realised through the same type of prosodic structure, they can be considered part of the same functional component: the interpersonal metafunction. Further, as they are both relatively independent of TRANSITIVITY, and TRANSITIVITY tends to have its own distinctive structure (a multivariate particulate structure, Halliday 1979), MOOD and MODALITY can be treated as forming a different metafunctional component to that of TRANSITIVITY (which is in the ideational: experiential metafunction). This line of argumentation shows that metafunctions can be derived from axis. Accordingly, this reasoning will be used throughout this paper to build a picture of the metafunctions of mathematical symbolism, which will in turn allow for a comparison of its functionality in relation to language.

3 Metafunctions in mathematics

This section will focus on the broad parameters of variation apparent in mathematical symbolism and will link these parameters with their structures.[2] It will show that there are indeed bundles of paradigmatically independent systems that maintain distinct structural organisations, but that these do not necessarily follow those of English precisely. This discussion is based on a more comprehensive description of mathematical symbolism presented in Doran (2017b).

3.1 Periodic structure and the textual component

We can begin by considering a simple mathematical statement, such as the equation:

(23) $$F = ma$$

This statement consists of two expressions, given by *F* and *ma*, which are linked by the Relator =. The first thing to notice here is that the two expressions can be swapped, without any change to their content meaning:

(24) $$ma = F$$

Both statements are perfectly grammatical and in some sense present the same relations. However they differ in ways that are important for the construction of texts. We can see this by looking at the statements in Text 1, from a senior high school physics textbook:

> Calculate how much weight a 50kg girl would lose if she migrated from the earth to a colony on the surface of Mars.

[2] More specifically, it will focus on a restricted register of mathematical symbolism roughly equivalent to elementary algebra. The corpus from which the study derives is of pre-calculus mathematics used in physics classrooms, textbooks and student work in a high school and university.

Answer

On the earth:

$$\begin{aligned} \boldsymbol{W}_{\text{earth}} &= m\boldsymbol{g}_{\text{earth}} \\ &= 50 \times 9.8 \\ &= 490 \text{ N downwards} \end{aligned}$$

On Mars:

$$\begin{aligned} \boldsymbol{W}_{\text{Mars}} &= m\boldsymbol{g}_{\text{earth}} \\ &= 50 \times 3.6 \\ &= 180 \text{ N downwards} \end{aligned}$$

Loss of weight = 490 – 180 = 310 N. But there is no loss of mass!

Text 1 de Jong et al. (1990: 249)

There are three groups of mathematical statements in this text, beginning with $\boldsymbol{W}_{\text{earth}}$, $\boldsymbol{W}_{\text{Mars}}$ and *Loss of weight*. Focusing first on the two beginning with $\boldsymbol{W}_{\text{earth}}$ and $\boldsymbol{W}_{\text{Mars}}$ we can see that each involves three lines moving through a similar sequence (known as a quantification genre, Doran 2017a). The first line begins with a single symbol on the left (e.g. $\boldsymbol{W}_{\text{Mars}}$) and a complex of pronumerical symbols on the right (e.g. $m\boldsymbol{g}_{\text{earth}}$). Following this, the second line elides the left symbol and replaces the right side with numbers (e.g.50×3.6). Finally, the third line continues the ellipsis on the left side and concludes the right side with a single number (e.g.180) as well as units (N, glossed as Newtons, the units of force) and a direction (downwards). The ellipsis of the left side of statements is a common pattern in mathematics.

If we fill in the ellipsis, these sections become:

$$\begin{aligned} \boldsymbol{W}_{\text{earth}} &= m\boldsymbol{g}_{\text{earth}} \\ \boldsymbol{W}_{\text{earth}} &= 50 \times 9.8 \\ \boldsymbol{W}_{\text{earth}} &= 490 \text{ N downwards} \end{aligned}$$

$$\begin{aligned} \boldsymbol{W}_{\text{Mars}} &= m\boldsymbol{g}_{\text{earth}} \\ \boldsymbol{W}_{\text{Mars}} &= 50 \times 3.6 \\ \boldsymbol{W}_{\text{Mars}} &= 180 \text{ N downwards} \end{aligned}$$

In this revised version, the symbols on the left (glossed as *weight on earth/Mars*) are repeated in each set of the equations. The right side, on the other hand, is changing. This pattern is also repeated in the statement in the final line of Text 1 *Loss of weight = 490 – 180 = 310 N*, the expression on the left also refers to weight, with the following expressions showing a progression of numbers. In comparison to the right side, on the left side of each set of statements, there is relative stability. This is a consistent feature across most texts in addition to the strong tendency for the left-side to contain only a single symbol (or at least fewer symbols than the right) and be non-numerical (i.e $\boldsymbol{W}_{\text{Mars}}$ as opposed to 50×3.6). This tendency is in spite of the fact that it is perfectly grammatical to swap the left and right side, or to have more symbols on the left than the right. For example, the following statements are all grammatical and in a sense, mean the same thing:

(25) $\boldsymbol{W}_{\text{earth}} = 50 \times 9.8$

(26) $50 \times 9.8 = \boldsymbol{W}_{\text{earth}}$

(27) $50 = \frac{\boldsymbol{W}_{\text{earth}}}{9.8}$

(28) $\frac{\boldsymbol{W}_{\text{earth}}}{9.8} = 50$

(29) $9.8 = \frac{\boldsymbol{W}_{\text{earth}}}{50}$

(30) $\frac{\boldsymbol{W}_{\text{earth}}}{50} = 9.8$

Despite the appearance of free variation in decontextualised examples, in text the probabilities of what will occur on the left and the right are significantly constrained. We can account for this by following O'Halloran's suggestion of the function of Theme for the left hand side of the statement (1999: 9). Like that for language, the Theme in mathematics tends toward stability (Fries 1981). It maintains the perspective through which the statement is being viewed and signals the relevancy of the statement to its co-text by indicating the symbols to which the text is orienting. Accordingly, the Theme tends not to be something completely new within the text, but something that has been mentioned previously, either in mathematics, language or other resources such as images. In Text 1, for example, the three Themes ($\boldsymbol{W}_{earth}$, W_{Mars}, and *Loss of weight*) all refer to weight, which is what the question asks to be calculated. Moreover, W_{earth} and W_{Mars} distinguish themselves by their subscripts that refer back to the circumstances of Location at the beginning of their section: *On the earth* and *On Mars* respectively. As we can see, the mathematical Themes are not pulled out of thin-air; they are related to the previous co-text. The Themes are used to emphasise the angle on the field that the statements are orienting to.

In contrast to the Theme, the right side of the statement expands the text and involves continual change, meaning it clearly performs a different function. To distinguish it from the Theme, we will use the term Articulation, which covers both the Relator (=) and the following expression. The Theme-Articulation structure of one of the sections in Text 1 is thus:

$\boldsymbol{W}_{\text{earth}}$	$= m\boldsymbol{g}_{\text{earth}}$
Theme	Articulation

$(\boldsymbol{W}_{\text{earth}})$	$= 50 \times 9.8$
Theme	Articulation

$(\boldsymbol{W}_{\text{earth}})$	$= 490$ N downwards
Theme	Articulation

The Articulation is an explicit elaboration of the Theme. Whereas the Theme orients the text to its field, the Articulation is more oriented toward genre staging. It shows the progression of a mathematical text from its beginning to its end. This Theme-Articulation structure is strongly tied to the unfolding genre-structure that the statements realise and is highly predictable from this (Doran 2017a).

So far we have only considered statements that involve two expressions such as $F = ma$. However the final equation in Text 1, *Loss of weight = 490 – 180 = 310 N*, shows that a statement can include more than two expressions, meaning that a simple allocation of one expression to Theme and one expression to Articulation does not work. Indeed, in principle, there can be any more than two expressions in a statement, such as the four in:

(31) $$\lambda = \frac{v}{f} = \frac{3\times10^8}{104.1\times10^6} = 2.9 \text{ m}$$

In practice, it is rare to get more than three or four expressions in a single statement. In fact, the unmarked choice is the minimal choice of two expressions, shown by all except the final statement in Text 1. Just like statements with two expressions, those with three or more have in principle free choice in the ordering of expressions. But again, in text, there is significant constraint on this ordering related to the genre structuring. Taking *Loss of weight = 490 –*

180 = 310 N as our example, there is a sense in which the third expression, *310 N*, follows on from and is in some sense logogenetically dependent on the second expression (*490 – 180*). This is due to the fact that the *310 N* is calculated from the subtraction of 180 from 490 in the second expression. By swapping these two expressions, this sense of an unfolding calculation would be lost; the generic structure would be out of sequence.

To account for this, each expression-Relator pair will be given a distinct function of Articulation. As there is the potential for indefinite expansion, each expression will be labeled Articulation_1, Articulation_2 etc. in sequence. This gives the analysis:

$Loss\ of\ weight$	$= 490 - 180$	$= 310\ N$
Theme	Articulation_1	Articulation_2

Following this, we can ask why, if a two-expression statement is the unmarked choice, would a text choose to expand its statement to three, four or more expressions? A possible solution is to consider this variation as another avenue for organising the information in the statement. We have said that the Theme remains relatively stable in order to maintain a particular angle on the field and to link it with its co-text. It thus provides a level of informational prominence in terms of the static orientation through which the text is read. On the other hand, the Articulation changes in order to provide new information that links with the unfolding of the text. One effect of expanding the statement is to background the non-final Articulations to a certain extent, and in doing so, make the final Articulation relatively salient or informationally prominent. That is, by expanding the statement, the final expression is presented as relatively salient or important for the unfolding text.

This function is similar to that of English New (often known as focus), where intonation is used to mark prominent new information through a significant pitch movement (for example the difference between having the pitch movement on the underlined elements in *I went to <u>the shops</u> yesterday* vs *I went to the shops <u>yesterday</u>*, Halliday and Greaves 2008). In the absence of intonation, the written mathematics appears to do this through lengthening the statement. This can be illustrated by Text 2.

The acceleration of a car which comes to rest in 5.4 seconds from a speed of 506 km/h is:

$$\text{average acceleration} = \frac{\text{change in speed}}{\text{time taken}}$$
$$= \frac{-506\ \text{km/h}}{5.4\ \text{s}} = -\,93.7\ \text{km/h/s}$$

This negative acceleration can be expressed as a deceleration of 93.7 km/h/s.

Text 2 Haire et al. (2000: 114)

In this text, the second statement, *(average acceleration)*= $\frac{-506km/h}{5.4\ s} = -\,93.7\ km/h/s$, involves three expressions. The final expression, $-\,93.7\ km/h/s$, gives the final result of the calculation while the second expression, $\frac{-506km/h}{5.4\ s}$, provides a step along the way to calculating it. Although the second expression ($\frac{-506km/h}{5.4\ s}$) is somewhat important in calculating the final result, it is not the point of the text; not much would be lost if it wasn't specified. In contrast, the final result shown by $-\,93.7\ km/h/s$ is crucial for the text's culmination. It thus makes sense that this expression is made informationally prominent.

Under this analysis, we can say that by extending the statement to include both expressions on the same line, this second expression is informationally backgrounded and quickly passed over, which in turn foregrounds the final result. Put another way, by extending the statement, the text establishes a small hierarchy of salience. Looked at from a discourse perspective, the relative importance of the final expression is reinforced by its repetition in the informationally prominent New of the final linguistic clause at the end of the text (the discourse semantic hyper-New, Martin 1992) - *This negative acceleration can be expressed as a deceleration of 93.7 km/h/s.*

This explanation means that there are two peaks of informational prominence in mathematical statements – the Theme and final Articulation – and that these potentially bookend a trough of backgrounded expressions. In this way, the information organisation resembles the periodic structure of the English clause related to the textual metafunction of

English discussed in the previous section. Figure 1 shows a system network that presents the broad choices for informational organisation of the statement. The network says that the system of THEME involves the choice of either a thematised or articulated expression. If the expression is articulated, then it can be either medial or final; if it is thematised then it can be either elided or explicit.

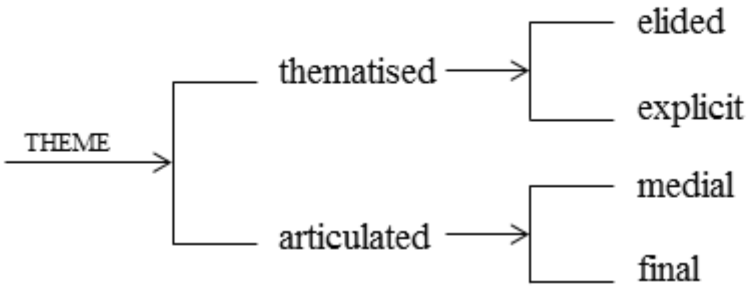

Figure 1 System of THEME in mathematics

3.2 Univariate structure and the logical component

The analysis of statements into Theme and Articulation gives an insight into the strong tendency for statements to organise expressions in terms of informational salience. However, as noted above this is only a tendency. Aside from this informational organisation, there is the potential for free variation as to what can be on the left or right. That is, both $\boldsymbol{W}_{\mathrm{earth}} = 50 \times 9.8$ and $50 \times 9.8 = \boldsymbol{W}_{\mathrm{earth}}$ are grammatical statements. Not only this, but there is a sense in which the two equations are to an extent the same. Indeed, for most people fluent in this type of mathematical symbolism, there would rarely be a need to distinguish the two. If we only account for the Theme^Articulation structure we would only be accounting for the difference between the two statements and we would miss their similarity.

To show their similarity, we can argue that from an alternative perspective, the two expressions perform the same function and are related by =. That is, we can label the two expressions as X and use subscripts for their sequencing, as in:

$\boldsymbol{W}_{\mathrm{earth}}$	$=$	50×9.8
X_1		X_2

By positing that both sides perform the same function, X, we can easily account for the similarity between $\boldsymbol{W}_{\text{earth}} = 50 \times 9.8$ and $50 \times 9.8 = \boldsymbol{W}_{\text{earth}}$. No matter the ordering, each performs the same function in relation to the other in both equations. From this perspective, this means that both $\boldsymbol{W}_{\text{earth}} = 50 \times 9.8$ and $50 \times 9.8 = \boldsymbol{W}_{\text{earth}}$ are the same.

With this addition, we thus have a means of accounting for the similarity of the two statements through their X_1, X_2 structure and accounting for their difference through their Theme-Articulation structure. For ease of reference, we will follow the convention of Halliday and Matthiessen (2014) of dropping the X and specifying only the subscript (1, 2) (though it must be stressed that 1, 2 indicates only sequence, it does not indicate any difference in function). We can therefore analyse the two tiers of structure as follows:

$\boldsymbol{W}_{\text{earth}}$	=	50×9.8
1		2
Theme	Articulation	

50×9.8	=	$\boldsymbol{W}_{\text{earth}}$
1		2
Theme	Articulation	

As there can be any number of expressions in a statement, the numbering can continue indefinitely, giving examples such as:

E_2	=	E_1	=	k
1		2		3
Theme	Articulation_1		Articulation_2	

λ	=	$\frac{v}{f}$	=	$\frac{3 \times 10^8}{104.1 \times 10^6}$	=	2.9 m
1		2		3		4
Theme	Articulation$_1$		Articulation$_2$		Articulation$_3$	

In each of these examples, the numbered expressions perform the same function. As discussed above, in Systemic Functional terms, this type of structure involving indefinite iterations of the same function is known as a univariate structure (Halliday and Matthiessen 2014).[3] This univariate structure maps onto the periodic Theme-Articulation structure discussed above.

It should be noted that all statements and Relator types, such as >, <, ≤, ≥ ≈, ∝ etc. are realised by a univariate structure and may be expanded indefinitely, not just equations with =. Examples include the three and four expression statements in:

(32) $a < r < b$

(33) $x \leq a \leq y \approx 0.5$

This means that all statements and Relator types can be analysed as having a univariate structure. The broadest distinction is between statements that can reverse their expressions without any change in Relator (known as symmetric statements and including =, ~, ≠,∝, ≡ and ≈), and those that order their expressions in terms of magnitude and thus require a change in Relator when reversing their expressions (known as magnitudinal statements and including >, <, ≥, ≤, ≪ and ≫). Aside from constraints to do with informational organisation, all statement types have in principle free choice of Theme. For magnitudinal Relators, this simply involves a shift from one Relator, such as >, to its counterpart <. This is somewhat similar to the active-passive distinction in English that maintains ideational meaning while shifting thematic structure.

[3] This univariate analysis is in contrast to O'Halloran's (1999, 2005) multivariate analysis of mathematical statements. For a detailed discussion of the two analyses see Doran (2017b).

The broad system network for statement type is given in Figure 2.[4] This network is read as follows. Statements involve two expressions (labeled 1 and 2) with a Relator between them. In addition, choices are to be made from both the systems of EXPANSION and RELATOR TYPE (shown through the brace {). Within RELATOR TYPE, there is a choice between either a symmetric (e.g. =, ~, ≠) or magnitudinal (e.g. >, <, ≥, ≤) Relator. In EXPANSION, there is a choice between either the basic statement involving two expressions or an expanded statement that adds another Relator and expression (+3). The loop indicates that this choice is iterative, allowing for any number of expressions.

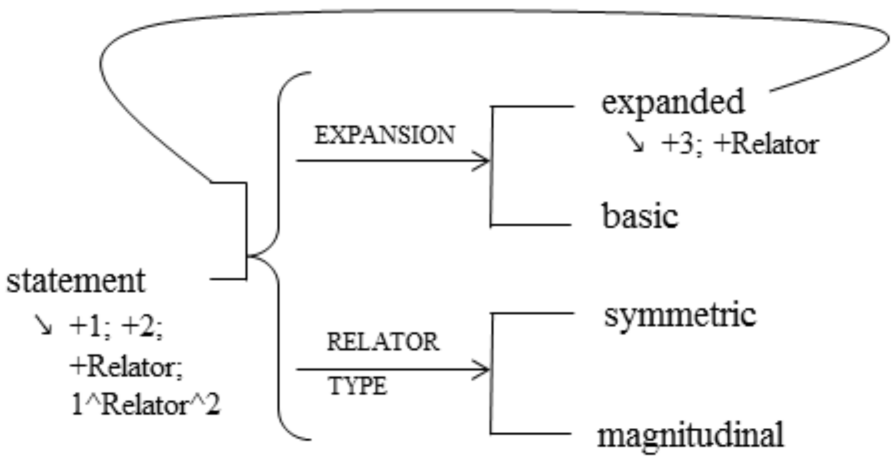

Figure 2 System network for mathematical statements

The importance of this network is that, as indicated above, there is free choice of Theme for any relator type and for any size of the statement. This indicates that there is relatively high paradigmatic independence between the informational systems governing choices of Theme and Articulation, and those governing Relator types and expansion. In addition to this, we have seen that these two sets of systems are realised through two distinct structural organisations. The THEME system is realised through a periodic structure and the RELATOR TYPE/EXPANSION systems are realised through a univariate structure.

Such paradigmatic independence and structural distinctions suggest these two systems may fall into two distinct metafunctional components. As the component involving the Theme-

[4] The realisation rules for this system are simplified here for readability, though the changes do not affect the overall argument of the paper. Doran (2017b) details a more explicit set of networks as well as more delicate options.

Articulation structure organises mathematical symbolism's information flow and is realised through a periodic structure, we can follow Halliday's description of English and consider it part of a textual metafunction of mathematics (Halliday and Matthiessen 2014). As the RELATOR TYPE and EXPANSION systems involve a univariate structure, we can also liken these to Halliday's model of English and consider them part of a logical metafunction of mathematics. This paradigmatic and syntagmatic organisation gives us our first evidence that mathematical symbolism is organised metafunctionally.

Before moving on, it is important to note that the logical component coordinates a number of areas of the grammar. For example within expressions, the organisation of symbols can also be modeled as an indefinitely iterative univariate structure. As the following equation shows, there can be one symbol in an expression (F) or there can be multiple symbols (0.5×3):

(34)

F	=	0.5	×	3
expression		expression		
symbol		symbol		symbol

When multiple symbols occur, they are related by a range of operators including multiplication ×, division ÷, addition +, subtraction -, roots √ and powers shown by a superscript, e.g. x^2. As O'Halloran (2005) points out, there is no limit to the number of relations and symbols that can occur within an expression and there is no need to postulate a central entity that performs a distinct function from all the others. This means that the organisation of symbols in expressions mirrors the organisation of expressions in statements by being indefinitely iterative. The expression on the right hand side of (35) for example, shows four symbols (3, z, 9 and 2) related by three operators ($\times$,$-$,$+$), while that of (36) involves sixteen symbols across three expressions:

(35) $$y = 3 \times z - 9 + 2$$

(36) $$r = 4\pi\varepsilon_0 \frac{n^2\hbar^2}{mZe^2} = \frac{n^2}{Z} a_0$$

To build a network of these choices, we can broadly distinguish between the arithmetic operators of addition (+), subtraction (−), multiplication (×) and division (÷), and a category that includes powers, roots and logarithms that, for want of a better term, we will call exponentiation. The system of EXPRESSION TYPE is given in Figure 3. This network indicates that with the insertion of a symbol into an expression (+ α), there is the choice of it being a single symbol in that expression (simple), or being related to other symbols (complex) through an arithmetic or exponentiation operation. As in Figure 2, the loop indicates the number of symbols in an expression is indefinitely expandable.

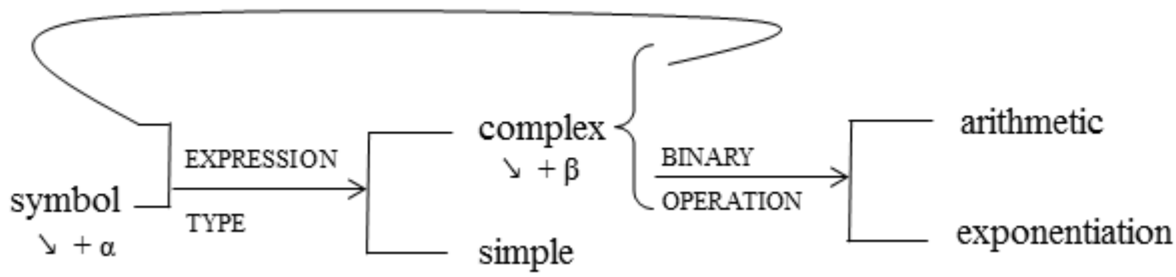

Figure 3 System network for mathematical symbols

Due to its structural similarity to the systems classified as part of the logical metafunction, it is useful to consider the complexing of symbols into expressions as also part of the logical metafunction.

3.3 Multivariate structure and the operational component

As a final step, there is one more area of the grammar that needs to be discussed. Although the logical and textual components coordinate much of the variation in mathematical symbolism, they do not account for it all. In particular, the internal structure of symbols follows a different set of organising principles that form their own component. We can see this by considering the use of subscripts to distinguish between different types of *E* (glossed as *Energy*) at the beginning of Text 3, a mathematical text written by a teacher on a white board in a high school physics class:

$$\Delta E_{emitted} = E_i - E_f \qquad E_i = \text{initial}$$

$$E_f = \text{final}$$

$$\begin{aligned} \Delta E_{3\rightarrow 2} &= -1.5 - -3.4 \\ &= 1.9 \text{ eV} \\ &= 1.9 \times 1.6 \times 10^{-19} \\ &= 3.04 \times 10^{-19} \quad \text{Joules} \end{aligned}$$

Text 3 High school physics class

This text is concerned with calculating the energy *E* when an electron moves between two levels in a hydrogen atom. To do this, it distinguishes between four different instances of energy using subscripts: E_i and E_f, glossed as initial and final energy during a transition, $E_{emitted}$, the energy emitted in a transition between two levels, and $E_{3\rightarrow 2}$, the energy emitted specifically in the transition between level 3 and level 2. As well as the subscripts, in the first and third line the use of the Greek character Δ modifies *E*. Δ is usually glossed as *change*, so that $\Delta E_{3\rightarrow 2}$ would be read as *the change in energy from 3 to 2*. Other types of modifications not shown in this text include the trigonometric functions sin, cos and tan, in $\sin\theta$, $\cos\theta$ and $\tan\theta$, factorials shown through !, as in 5! and numerous others. These modifiers form a valuable component of the discourse, performing a host of different functions within texts.

Each modifier has a number of features that distinguish them from the heads that they are dependent on. The first is that they cannot occur on their own in an expression. That is, each the following equations are not possible:

(37) $*\Delta = y$

(38) $*\text{sin} = \text{y}$

(39) $*\ \ _2 = y$

Second, operations such as multiplication, division, addition etc. cannot occur between the modifier and its head:

(40) $*\Delta \times y$

(41) $*\text{sin} \times \text{y}$

(42) $*y_{\times 2}$

These two characteristics distinguish modifiers from the symbols they modify, such as x, y, *2*, π etc. This sets up two distinct functions: those that can sit on their own in an expression and can be related to other symbols through operations such as multiplication, and those that cannot. Illustrating this with the subscript relation, E_i, we will call E a Quantity, and the subscript a Specifier. Thus we would analyse E_i as Quantity^Specifier. Quantity will be used for any symbol that can enter into an operation such as multiplication (×), addition (+), division (÷) etc. Other modifiers include the change operator Δ and the factorial !, which will all be grouped under the general function Operation (detailed arguments for this distinction are given in Doran 2017b).

The fact that the Quantity and its modifiers perform distinct functions is an important point for our understanding of metafunctions. This division in functionality differs from the univariate structures associated with organising symbols into expressions and expressions into statements. Indeed, in addition to performing different functions, these functions are not all indefinitely iterative. We cannot have two Quantities in a row, such as *Δxy (where their sequence does not indicate multiplication), and although possible, it is unusual for modifiers to be repeated indefinitely (e.g. $\Delta\Delta\Delta x$ would be odd, at best). This structure is therefore best modeled not as an indefinitely iterative univariate structure, but as a multivariate structure (Halliday and Matthiessen 2014). As discussed above, multivariate structures are those that involve multiple functions that typically occur only once, and in language are associated with the experiential component of the ideational metafunction.

As well as the structural distinction between this internal organisation of symbols and the other systems discussed so far, the systems are also paradigmatically independent of each other. For example, any configuration of Quantity + modifier (either Specifier or Operation) can be complexed with any other. Examples we have already seen include a_0 and ε_0 being complexed with other symbols in:

(43) $$r = 4\pi\varepsilon_0 \frac{n^2\hbar^2}{mZe^2} = \frac{n^2}{Z} a_0$$

Another example is the Operation cosine (cos) modifying x, in:

(44) $$y \cos x \approx 0.5$$

In addition, a modified symbol can become Theme or Articulation just like any other symbol:

(45) $0.5 \approx y \cos x$

(46) $\cos x \approx \frac{0.5}{y}$

(47) $\frac{0.5}{y} \approx \cos x$

(48) $y \approx \frac{0.5}{\cos x}$

(49) $\frac{0.5}{\cos x} \approx y$

As there is both paradigmatic independence and syntagmatic difference between the internal organisation of symbols and the systems in the logical and textual components, a case holds for these systems to form their own metafunctional component. The multivariate structures in this component are similar to those within the experiential metafunction in English. However notionally, the experiential metafunction is concerned with construing our experience of the outside world (Halliday 1979). For example in English, the experiential system of TRANSITIVITY divides the clause into material, mental and relational clauses (Matthiessen 1995), broadly construing the realms of doing, thinking and being. Although in an axial description such as developed here, the notional 'meanings' of categories are not privileged, they are helpful when labeling. It is difficult to reconcile the variation internal to symbols as in some way construing our experience of the outside world. In this sense, the label 'experiential' is somewhat awkward. Indeed aside from their multivariate structure, there is little in common between this component and the experiential component in language. Rather than construing experience, this component is more concerned with operating on symbols. For instance, the change Operation Δ modifies a variable such that it specifies its change between two points. Symbolically, this is given by: $\Delta x = x_2 - x_1$ (glossed as *the change in x equals the value of x at point 2 minus the value of x at point 1*), e.g. if $x_2 = 5$ and $x_1 = 3$, then $\Delta x = x_2 - x_1 = 5 - 3 = 2$. Thus, to more easily capture this component's nature, it will be called the *operational* component.

Although they present distinct subtypes of structure – univariate and multivariate – as the logical and the operational components both broadly utilise particulate structures they can be grouped together as parts of the ideational metafunction. The systems involved in the operational component are shown in Figure 4 as the system of MODIFICATION. In this figure, MODIFICATION is shown as paradigmatically independent of the logical systems of EXPRESSION TYPE and the textual systems of THEME.[5]

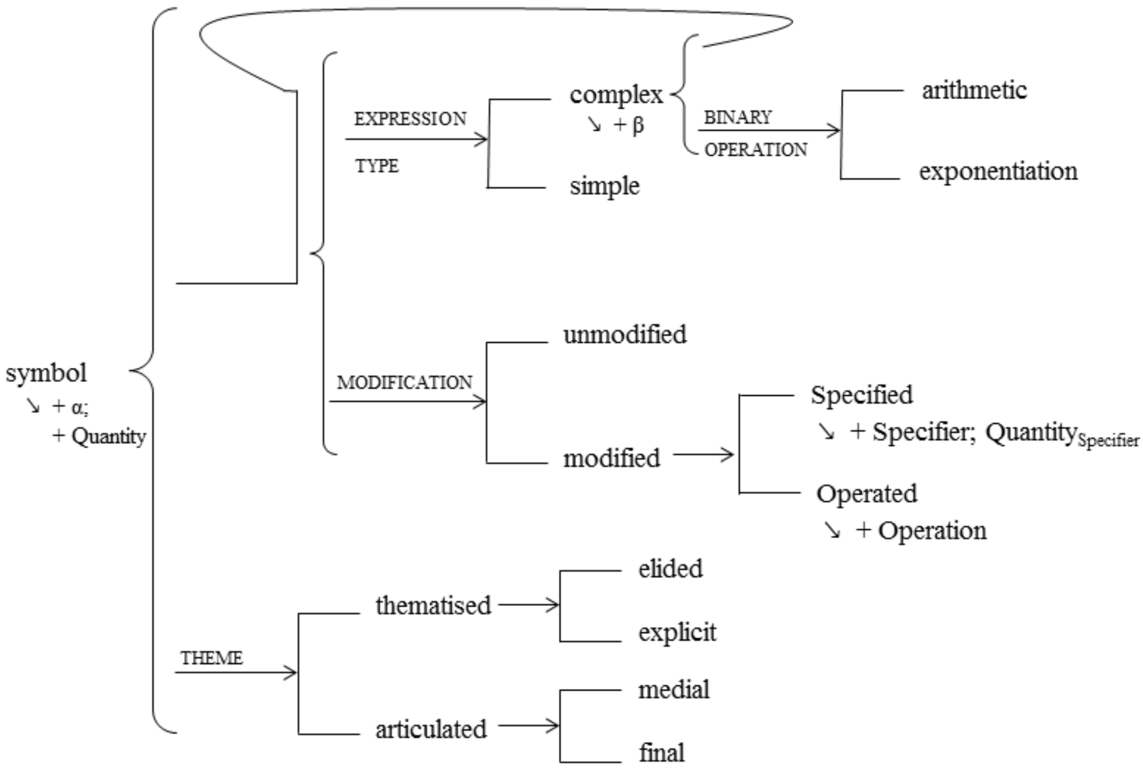

Figure 4 Expanded system network for mathematical symbols

3.4 Against an interpersonal component in mathematics

The description in this paper has made a point of not assuming that metafunctions will neatly transfer across from English into mathematics and other semiotic resources. This has been done in order to more productively understand the intrinsic functionality of mathematics in relation to language. Rather, it has sought to derive metafunctions from the more primitive Systemic Functional dimensions of the paradigmatic and syntagmatic axes. Through this method we have been able to build an idea of the architecture of mathematical symbolism as organised through three distinct metafunctional components: the logical, the textual and the operational. What is notably missing in comparison to the Systemic Functional model of language is a distinct interpersonal component. Indeed from an axial perspective, there

[5] The system shown in Figure 2 that organise expressions into statements occurs at a higher level than these systems. As the level of symbol shown in Figure 4 contains systems from each of the three metafunctions, it is used to illustrate their independence here.

appears not to be any evidence to propose a distinct interpersonal component in mathematics. The three components outlined so far, the logical, operational and textual, account for all the systems in the grammar (for full details see Doran 2017b). There are no apparent systems realised through a prosodic structure like those for MOOD and MODALITY discussed in Section 2, nor are there other bundles of systems paradigmatically independent of those within the three components presented so far. Indeed looking notionally, there are no systems that appear to allow for typically 'interpersonal' meanings involved with negotiation through dialogue such as a MOOD or NEGOTIATION system nor those that give evaluative meanings of APPRAISAL or the power and solidarity dimensions associated with Vocatives and other naming practices. Without paradigmatic independence or distinct syntagmatic structure, there is no reason to suggest a distinct interpersonal component in mathematics.

This description notably contrasts with that of O'Halloran (1999, 2005) which presumes an interpersonal component and develops its description accordingly. It is thus pertinent to consider some of the important issues raised by O'Halloran regarding the possibility for an interpersonal component in mathematical symbolism. First, O'Halloran points out that there are some similarities in meaning between certain Relators in mathematics and interpersonal constructions in English. In particular, she suggests a system of POLARITY to distinguish between the positive polarity of = and the negative polarity of ≠ (often glossed as *not equal to*) (2005: 100, 155). In systemic functional studies, POLARITY in English is generally considered an interpersonal system (Martin 1983, though not without some contention, see Halliday 1978:132 where it is treated as experiential and Fawcett 2008 where it is its own functional component), thus O'Halloran considers the distinction between = and ≠ to also be interpersonal in mathematics. Along similar lines, we could consider Relators such as ~ and ≈, both glossed as *approximately equal to*, to give some sort of gradability meaning associated with GRADUATION in English, another interpersonal system (Martin and White 2005). Arguing along these lines, these Relators could therefore also form part of an interpersonal component. However developing an interpersonal component based on vague similarities in meaning goes against the principled description in this chapter. Arguments along these lines rely on notional reasoning that analogises from English. In contrast, if these Relators are looked at in terms of their paradigmatic and syntagmatic organisation, we see that each of them, ≠, =, ~ and ≈ are entirely dependent on choosing a symmetric statement and not a magnitudinal statement (i.e. they are not in the same group as the >, <, ≤, ≥ etc.). This distinction between symmetric and magnitudinal statements was classified as part of the logical component. Thus, the choice between $\neq$, $=$, $\sim$ or $\approx$ is not paradigmatically independent from the rest of the logical component. Further, they are not realised by a distinct structure from the logical component's univariate structure. Thus, these choices do not form their own distinct metafunctional component; they are firmly within the logical component. They do, however, suggest that the meanings of polarity and graduation have been in some sense 'ideationalised' when translated into mathematics.

An interpretation along these lines allows an understanding of the quantification of certainty through probability, statistics and measurement errors, in relation to linguistic modality. As O'Halloran (2005: 115) states, "In mathematics, choices for MODALITY in the form of probability may be realised through symbolic statements or measures of probability; for example, levels of significance: $p < 0.5$ (where the notion of uncertainty is quantified) and different forms of approximations." Viewed from the description developed in this chapter, the meanings of modality have been ideationalised in a similar way to the polarity and graduation meanings discussed above. What would be expressed interpersonally in language through graded modalisation of probability (e.g. *The hypothesis is probably true*), is expressed through an ideationally organised mathematical statement (such as through p-values: e.g. $p < 0.05$ used to determine the likelihood of a hypothesis being true or false). What is interpersonal in language can be seen as quantified and ideationalised in mathematics. Although statistical mathematics is not studied in detail in this description, it is possible that this system has developed largely to ideationalise and more firmly particularise what would in language be fuzzy interpersonal measures of modalisation.

A second important observation by O'Halloran regards interactions between mathematics and language in instances such as $Let\ x = 2$. The use of *Let* before the mathematical statement indicates the construction is a command (demanding goods and services). However without the *Let*, the mathematical statement is arguably more similar to a linguistic statement (giving information). Thus the *Let* in some sense affects the speech function of the mathematical statement, giving it variability in interpersonal meaning. In regards to whether this constitutes evidence for an interpersonal component in mathematics, the grammar developed in this description only considers mathematics as an isolated system; it does not look at the interaction of mathematics and language. From this perspective, the introduction of language into the statement is immaterial to a discussion of the functionality *internal* to the system of mathematics. However it does highlight an important challenge that has yet to be fully solved. The introduction of language appears to contextualise the mathematics transposing the speech-functional meanings from language across to mathematics. This raises the question of how Systemic Functional and Social Semiotics are to model metafunctionality across inter-semiotic systems, or indeed across semiosis in general. Crucially, arguing that mathematics does not have an interpersonal component internal to the system does not preclude the possibility that mathematics occurs in texts with resources that do engender interpersonal meaning. More broadly speaking, with multimodal texts, the various functionalities of each resource are likely to contextualise one another. In the case of $Let\ x = 2$, as mathematics does not have the ability to distinguish speech functions, it appears that language is being 'imported' as necessary to make these meanings. The distinction in functionality between the two is being utilised, whereby the interpersonal in language is used for meanings that mathematics cannot make.

In short, the lack of an internally motivated interpersonal component in mathematics does not preclude it from being involved in interpersonal meaning in a multimodal text. What it does suggest is that mathematics cannot produce variations in interpersonal meaning of its own accord; it must do so in interaction with other semiotic resources. In addition, it suggests that interpersonal resources are likely to be ideationalised when translated into mathematics. It is possible that this ideationalising feature is a large reason for its powerful role in academic disciplines (Doran 2017a), which gives an insight into its differences to language.

4 Intrinsic functionality, theory and description.

We began this paper by asking what the similarities and differences between mathematics and language are. This question was then approached through the dimension of metafunction as conceived in Systemic Functional and Social Semiotic studies. However it was argued that assuming metafunctions occur across semiosis risks homogenising descriptions and watering down differences in the specific functionality of mathematics and other semiotic resources. Thus in response, this paper suggested that metafunction could be more fruitfully understood by considering the paradigmatic and syntagmatic organisation of mathematics. Through this method, we have seen that mathematical symbolism is indeed organised metafunctionally, but that the metafunctional configuration is not precisely the same as for language. Where language is organised around four functional components – termed the experiential, logical, textual and interpersonal – mathematical symbolism is organised around only three – the logical, textual and operational. Significantly, there seemed to be no evidence for an interpersonal component similar to that of English; there appeared no additional set of systems independent from those in the logical, operational and textual, and no systems that were realised through a prosodic structure.

This pushes the current Systemic Functional and Social Semiotic understandings of semiosis. In Systemic Functional Linguistics, a distinction is often made between theoretical and descriptive categories (Halliday 1992b, Caffarel et al. 2004). As Halliday argues:

> Theoretical categories are, by definition, general to all languages: they have evolved in the construction of a general linguistic theory. They are constantly being refined and developed as we come to understand more about language; but they are not subject to direct verification. A theory is not proved wrong; it is made better – usually step by step, sometimes by a fairly catastrophic change.
>
> Descriptive categories are in principle language-specific: they have evolved in the description of particular languages. Since we know that all human languages have much in common, we naturally use the descriptive categories of one language as a guide when working in another. But if a descriptive category named “clause” or “passive” or “Theme” is used in describing, say, both English and Chinese, it is redefined in the case of each language…
>
> So, for example, while “system” itself is a theoretical category, each instance of a system, such as “mood”, is a descriptive category. Similarly, “option” (or “feature”) in a system is a theoretical category, while each particular instance of an option, like “indicative” or “declarative”, is descriptive. (Halliday 1992b: 11).

In Systemic Functional Linguistics, the four-way distinction of metafunction is generally considered to be a theoretical category that is assumed to occur for all languages (Halliday 1992b; Caffarel et al. 2004). In contrast, this paper has argued that this assumption is not necessarily useful across all of semiosis. In this sense, viewed from the perspective of Systemic Functional *Semiotics* – the theory of semiosis in general – metafunctions are not theoretical categories general to all semiosis but rather descriptive categories that are specific to each semiotic resource and must be defined and justified in each case. What was assumed as theoretical categories in this paper were those associated with axis, such as system (paradigmatic axis), structure (syntagmatic axis), function, class, option (or feature) and realisation (Caffarel et al. 2004, Martin 2011).

For mathematical symbolism, by shifting metafunction to a descriptive category, we can begin to grasp its broad functionality in comparison with language. Whereas language is significantly geared toward interpersonal concerns – enacting and negotiating social relations – mathematics is not. Rather, mathematics' system is largely organised through its ideational component. Indeed when translated into mathematics, meanings that would be organised interpersonally in language become ideationalised. Although not discussed here in detail, its logical and operational components dominate the overall configuration of the grammar. They maintain distinct level hierarchies in the grammar and organise the vast majority of possible variation apparent in mathematics. This has given it a specific functionality whereby large swathes of ideational relations can be related relatively efficiently, which in turn has ensured its usefulness for construing the highly technical knowledge of the sciences (O'Halloran 2005, Doran 2017a).

This discussion allows us to return to our original question – is mathematics a language? If, like most Systemic Functional and Social Semiotic studies, languages are conceived as having four broad functional components that organise both their internal architecture and their external functionality, then the lack of an interpersonal component in mathematics suggests classifying it as a language is not productive. Unlike language, mathematics does not offer ways of negotiating through dialogue, presenting evaluative or affective states or construing degrees of social status or contact. However as we have seen, mathematics does involve significant structural and systemic organisation that is very similar to language. In this sense, although not a language in the most precise sense, mathematics does, to an extent, maintain some similar functionalities. More broadly, what this paper has illustrated is that the broad architecture of semiotic resources can be probed using the Systemic Functional primitive of axis by exploring its paradigmatic and syntagmatic organisation. This potentially offers a means through which the functionality of semiotic resources can be investigated in a principled manner, without assuming macrotheoretical categories developed for language apply across all semiosis.

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